

$$u_r = -\frac{K_T P}{3} r - \frac{P}{4\mu} \frac{a^3}{r^2}$$

where μ is the shear modulus. The strain components, obtained from the appropriate derivatives of the displacement field, are

$$e_{rr} = -\frac{K_T P}{3} + \frac{P}{2\mu} \frac{a^3}{r^3},$$

$$e_{\theta\theta} = -\frac{K_T P}{3} - \frac{P}{4\mu} \frac{a^3}{r^3},$$

and

$$e_{\phi\phi} = -\frac{K_T P}{3} - \frac{P}{4\mu} \frac{a^3}{r^3}.$$

The deviatoric strain, e_{ij}^d , is obtained by subtracting the hydrostatic strain. This is the only part that contributes to the magnetoelastic energy. In local Cartesian coordinates, the deviatoric strain tensor is

$$e_{ij}^d = \frac{P}{4\mu} \frac{a^3}{r^3} \left[3 \frac{x_i x_j}{r^2} - \delta_{ij} \right].$$

Using this expression for the strain in the magnetoelastic energy,

$$\begin{aligned} \epsilon_{me} = & b_1 (\alpha_1^2 e_{11} + \alpha_2^2 e_{22} + \alpha_3^2 e_{33}) + 2b_2 (\alpha_1 \alpha_2 e_{12} + \\ & \alpha_2 \alpha_3 e_{23} + \alpha_3 \alpha_1 e_{31}), \end{aligned}$$

and using the average values in Table 1 where

$$\frac{x_i x_j}{r^2} = n_i n_j,$$

one obtains

$$\epsilon_{me} = \frac{3P}{4\mu} B \frac{a^3}{r^3} \cos^2(\psi + \theta)$$

where

$$B = \frac{2}{5} b_1 + \frac{3}{5} b_2.$$

This is the magnetoelastic energy in Equation (3.16).